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A Kind Tripartite Entangled State Representation and Its Application in Quantum Teleportation

Cui-hong Lv · Hong-yi Fan

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Abstract We discuss entanglement for a tripartite system by setting up a new state vector representation $|p,\chi_1,\chi_2\rangle$ in three-mode Fock space. The Schmidt decomposition of $|p,\chi_1,\chi_2\rangle$ is presented and its application in teleporting a bipartite entangled state or a two-mode squeezed state to a pair of receivers is analyzed.

Keywords Quantum teleportation · Quantum entanglement · New tripartite entangled state

1 Introduction

The concept of entanglement, as first pointed out by Einstein, Podolsky and Rosen (EPR) [1] in their famous paper arguing the incompleteness of quantum mechanics, plays a key role in understanding some fundamental problems in quantum mechanics and quantum optics. EPR's idea has stimulated us to construct various entangled state representations due to their potential uses in quantum computation [2, 3], quantum teleportation [4–9] and so on. In an entangled quantum state with continuous variables, measurements performed on one part of the system provides information on the remaining part; this is now known to be a basic feature of quantum mechanics, though it seems weird [1, 10–12]. In Ref. [4], Bernett et al. have suggested that by virtue of entanglement, it is possible to transfer the quantum state of a particle onto another particle provided one does not get any information about the state in the course of this transformation. The experimental quantum teleportation of a discrete variable and a continuum variable was successfully done by the Zeilinger group [7] and the Kimble group [8], respectively. The theoretical analysis of the teleportation of continuous quantum states was first made by Vaidman [5].

Multipartite entanglement, the entanglement shared by more than two particles, seems much more difficult to handle both theoretically and experimentally [13, 14]. In this paper we shall discuss entanglement for a tripartite system by setting up a new state vector

C. Lv (✉) · H. Fan
Department of Material Science and Engineering, University of Science and Technology of China, Hefei, Anhui 230026, China
e-mail: lvcuihong@mail.ustc.edu.cn

$|p,\chi_1,\chi_2\rangle$ (the eigenvector of the three compatible operators: $(X_1 - X_3)$, $(X_1 + X_2 + X_3)$ and $(P_1 - 2P_2 + P_3)$) in three-mode Fock space, where X_j, P_j $(j = 1, 2, 3)$ are coordinate and momentum, respectively. By doing Schmidt decomposition for $|p,\chi_1,\chi_2\rangle$ its entanglement will exhibit explicitly. Then, we shall apply this three-mode entangled state as a quantum channel to quantum teleporting a bipartite entangled state or a two-mode squeezed state to a pair of receivers. This work is organized as follows. In Sect. 2, we briefly introduce the tripartite entangled state representation in the Fock space. Then in Sect. 3, we derive the Schmidt decomposition of $|p,\chi_1,\chi_2\rangle$ so that its entanglement can be seen more clearly. The application of $|p,\chi_1,\chi_2\rangle$ to quantum teleporting an entangled state or a two-mode squeezed state to a pair of receivers is investigated in Sect. 4.

2 The Common Eigenstates of $X_1 - X_3$, $X_1 + X_2 + X_3$ and $P_1 - 2P_2 + P_3$

We find that the common eigenvector, denoted by $|p,\chi_1,\chi_2\rangle$, of the three compatible operators $(X_1 - X_3)$, $(X_1 + X_2 + X_3)$ and $(P_1 - 2P_2 + P_3)$, in three-mode Fock space has the form,

$$\begin{aligned}|p,\chi_1,\chi_2\rangle = \frac{1}{\sqrt{6}\pi^{3/4}}\exp\Big[&-\frac{1}{12}\left(p^2+3\chi_1^2+2\chi_2^2\right)+\frac{\sqrt{2}}{6}ip\left(a_1^\dagger-2a_2^\dagger+a_3^\dagger\right)\\&+\frac{\sqrt{2}}{2}\chi_1\left(a_1^\dagger-a_3^\dagger\right)+\frac{\sqrt{2}}{3}\chi_2\sum_{j=1}^{3}a_j^\dagger-\frac{1}{6}\left(2a_1^{\dagger 2}-a_2^{\dagger 2}+2a_3^{\dagger 2}\right)\\&-\frac{1}{3}\left(2a_1^\dagger a_2^\dagger-a_1^\dagger a_3^\dagger+2a_2^\dagger a_3^\dagger\right)\Big]|000\rangle,\end{aligned} \tag{1}$$

where p, χ_1 and χ_2 are real numbers, a_j, $a_j^\dagger$ $(j = 1, 2, 3)$ are bose operators satisfying $[a_j, a_k^\dagger] = 1$, $|000\rangle$ is the three-mode vacuum state. In fact, respectively operating a_j on $|p,\chi_1,\chi_2\rangle$ gives

$$a_1|p,\chi_1,\chi_2\rangle = \frac{1}{3}\left[\frac{\sqrt{2}}{2}(ip+3\chi_1+2\chi_2)-\left(2a_1^\dagger+2a_2^\dagger-a_3^\dagger\right)\right]|p,\chi_1,\chi_2\rangle, \tag{2}$$

$$a_2|p,\chi_1,\chi_2\rangle = \frac{1}{3}\left[\sqrt{2}(-ip+\chi_2)+\left(a_2^\dagger-2a_1^\dagger-2a_3^\dagger\right)\right]|p,\chi_1,\chi_2\rangle, \tag{3}$$

$$a_3|p,\chi_1,\chi_2\rangle = \frac{1}{3}\left[\frac{\sqrt{2}}{2}(ip-3\chi_1+2\chi_2)+\left(a_1^\dagger-2a_2^\dagger-2a_3^\dagger\right)\right]|p,\chi_1,\chi_2\rangle. \tag{4}$$

It then follows from

$$X_j = \frac{a_j + a_j^\dagger}{\sqrt{2}}, \qquad P_j = \frac{a_j - a_j^\dagger}{i\sqrt{2}} \tag{5}$$

that

$$(X_1 - X_3)|p,\chi_1,\chi_2\rangle = \chi_1|p,\chi_1,\chi_2\rangle, \tag{6}$$

$$(X_1 + X_2 + X_3)|p,\chi_1,\chi_2\rangle = \chi_2|p,\chi_1,\chi_2\rangle, \tag{7}$$

$$(P_1 - 2P_2 + P_3)|p,\chi_1,\chi_2\rangle = p|p,\chi_1,\chi_2\rangle. \tag{8}$$

By virtue of the technique of integral within an ordered product (IWOP) of operators [15–18] and the normal product form of the three-mode vacuum state projector

$$|000\rangle\langle 000| = :\exp\left(-a_1^\dagger a_1 - a_2^\dagger a_2 - a_3^\dagger a_3\right):, \tag{9}$$

where :: denotes normal ordering, we can easily prove that the set of $|p, \chi_1, \chi_2\rangle$ spans a complete set, i.e.,

$$\begin{aligned}
&\iiint dp\, d\chi_1\, d\chi_2 |p, \chi_1, \chi_2\rangle\langle p, \chi_1, \chi_2| \\
&\quad = \frac{1}{6\pi^{3/2}} \iiint dp\, d\chi_1\, d\chi_2 :\exp\bigg[-\frac{1}{6}\left(p^2 + 3\chi_1^2 + 2\chi_2^2\right) \\
&\qquad + \frac{\sqrt{2}}{6} ip\left(a_1^\dagger - 2a_2^\dagger + a_3^\dagger - a_1 + 2a_2 - a_3\right) \\
&\qquad + \frac{\sqrt{2}}{2}\chi_1\left(a_1^\dagger - a_3^\dagger + a_1 - a_3\right) + \frac{\sqrt{2}}{3}\chi_2\left(a_1^\dagger + a_2^\dagger + a_3^\dagger + a_1 + a_2 + a_3\right) \\
&\qquad - \frac{1}{6}\left(2a_1^{\dagger 2} - a_2^{\dagger 2} + 2a_3^{\dagger 2} + 2a_1^2 - a_2^2 + 2a_3^2\right) - a_1^\dagger a_1 - a_2^\dagger a_2 - a_3^\dagger a_3 \\
&\qquad - \frac{1}{3}\left(2a_1^\dagger a_2^\dagger - a_1^\dagger a_3^\dagger + 2a_2^\dagger a_3^\dagger + 2a_1 a_2 - a_1 a_3 + 2a_2 a_3\right)\bigg]: \\
&\quad = :e^0: = 1.
\end{aligned} \tag{10}$$

Using (6)–(8), we can also calculate

$$\begin{aligned}
\langle p', \chi_1', \chi_2'|(X_1 - X_3)|p, \chi_1, \chi_2\rangle &= \chi_1'\langle p', \chi_1', \chi_2'|p, \chi_1, \chi_2\rangle \\
&= \chi_1\langle p', \chi_1', \chi_2'|p, \chi_1, \chi_2\rangle,
\end{aligned} \tag{11}$$

$$\begin{aligned}
\langle p', \chi_1', \chi_2'|(X_1 + X_2 + X_3)|p, \chi_1, \chi_2\rangle &= \chi_2'\langle p', \chi_1', \chi_2'|p, \chi_1, \chi_2\rangle \\
&= \chi_2\langle p', \chi_1', \chi_2'|p, \chi_1, \chi_2\rangle,
\end{aligned} \tag{12}$$

$$\begin{aligned}
\langle p', \chi_1', \chi_2'|(P_1 - 2P_2 + P_3)|p, \chi_1, \chi_2\rangle &= p'\langle p', \chi_1', \chi_2'|p, \chi_1, \chi_2\rangle \\
&= p\langle p', \chi_1', \chi_2'|p, \chi_1, \chi_2\rangle,
\end{aligned} \tag{13}$$

which tell us that $|p, \chi_1, \chi_2\rangle$ is orthogonal

$$\langle p', \chi_1', \chi_2'|p, \chi_1, \chi_2\rangle = \delta(p' - p)\delta(\chi_1' - \chi_1)\delta(\chi_2' - \chi_2). \tag{14}$$

Thus $|p, \chi_1, \chi_2\rangle$ is qualified to be a continuous Bell basis, and to make up a quantum mechanical representation, it can be taken as an ideal quantum channel, and $|p, \chi_1, \chi_2\rangle$ $\langle p, \chi_1, \chi_2|$ as an ideal quadrature phase measurement basis.

On the other hand, the common eigenvector of $\{(P_1 - P_3), (P_1 + P_2 + P_3), (X_1 - 2X_2 + X_3)\}$, the conjugate state of $|p, \chi_1, \chi_2\rangle$, is found to be

$$\begin{aligned}
|\chi, p_1, p_2\rangle = &\frac{1}{\sqrt{6}\pi^{3/4}} \exp\bigg[-\frac{1}{12}\left(\chi^2 + 3p_1^2 + 2p_2^2\right) + \frac{\sqrt{2}}{2} ip_1\left(a_1^\dagger - a_3^\dagger\right) \\
&+ \frac{\sqrt{2}}{3} ip_2\left(\sum_{j=1}^{3} a_j^\dagger\right) + \frac{\sqrt{2}}{6}\chi\left(a_1^\dagger - 2a_2^\dagger + a_3^\dagger\right) + \frac{1}{6}\left(2a_1^{\dagger 2} - a_2^{\dagger 2} + 2a_3^{\dagger 2}\right)
\end{aligned}$$

$$+\frac{1}{3}\left(2a_1^\dagger a_2^\dagger - a_1^\dagger a_3^\dagger + 2a_2^\dagger a_3^\dagger\right)\Bigg]|000\rangle, \tag{15}$$

which obeys the eigenvector equations

$$(P_1 - P_3)|\chi, p_1, p_2\rangle = p_1|\chi, p_1, p_2\rangle, \tag{16}$$

$$(P_1 + P_2 + P_3)|\chi, p_1, p_2\rangle = p_2|\chi, p_1, p_2\rangle, \tag{17}$$

$$(X_1 - 2X_2 + X_3)|\chi, p_1, p_2\rangle = \chi|\chi, p_1, p_2\rangle. \tag{18}$$

The term "conjugate" originates from the canonical conjugate commutators of the operators appearing in (6)–(8) and those in (16)–(18), i.e.,

$$[X_1 - X_3, P_1 - P_3] = 2i, \tag{19}$$

$$[X_1 + X_2 + X_3, P_1 + P_2 + P_3] = 3i, \tag{20}$$

$$[P_1 - 2P_2 + P_3, X_1 - 2X_2 + X_3] = 4i. \tag{21}$$

$|\chi, p_1, p_2\rangle$ is complete and orthogonal too,

$$\iiint d\chi\, dp_1\, dp_2 |\chi, p_1, p_2\rangle\langle\chi, p_1, p_2| = 1, \tag{22}$$

$$\langle\chi', p_1', p_2'|\chi, p_1, p_2\rangle = \delta(\chi' - \chi)\delta(p_1' - p_1)\delta(p_2' - p_2). \tag{23}$$

3 The Schmidt Decomposition of $|p, \chi_1, \chi_2\rangle$

As is well known, by entanglement [19, 20] one means that a two-particle state does not factor into a product of single-particle states, but is a sum of at least two terms, each of which is a product. For example, EPR entangled state $|\eta\rangle$ [21], which is the simultaneous eigenstate $|\eta\rangle$ of two particles' relative position $X_1 - X_2$ and total momentum $P_1 + P_2$, can be decomposed into

$$|\eta\rangle = e^{-i\eta_1\eta_2/2}\int_{-\infty}^{\infty} dx|x\rangle_1 \otimes |x - \eta_1\rangle_2 e^{i\eta_1 x} \tag{24}$$

where the coordinate eigenvectors $|x\rangle_j$ (the coordinate eigenstate of X_j, $j = 1, 2$) are defined as

$$|x\rangle_j = \pi^{-1/4}\exp\left(-\frac{1}{2}x^2 + \sqrt{2}xa_j^\dagger - \frac{1}{2}a_j^{\dagger 2}\right)|0\rangle_j. \tag{25}$$

For revealing entanglement involved in $|p, \chi_1, \chi_2\rangle$, we consider the following one-fold Fourier integration

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} dp e^{-ixp}|p, \chi_1, \chi_2\rangle$$
$$= \frac{1}{\pi^{3/4}\sqrt{2\pi}}\exp\Bigg[-\frac{1}{6}\left(a_1^\dagger - 2a_2^\dagger + a_3^\dagger - 3\sqrt{2}x\right)^2 - \frac{1}{4}\chi_1^2 - \frac{1}{6}\chi_2^2$$

$$+\frac{\sqrt{2}}{2}\chi_1\left(a_1^\dagger - a_3^\dagger\right) + \frac{\sqrt{2}}{3}\chi_2\left(\sum_{j=1}^{3} a_j^\dagger\right) - \frac{1}{6}\left(2a_1^{\dagger 2} - a_2^{\dagger 2} + 2a_3^{\dagger 2}\right)$$
$$\left. - \frac{1}{3}\left(2a_1^\dagger a_2^\dagger - a_1^\dagger a_3^\dagger + 2a_2^\dagger a_3^\dagger\right)\right]|000\rangle. \tag{26}$$

By comparison with the expression of the coordinate eigenstate (25), we see that

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} dp e^{-ixp}|p, \chi_1, \chi_2\rangle$$
$$= \frac{1}{\sqrt{2\pi}}\left|x + \frac{1}{2}\chi_1 + \frac{1}{3}\chi_2\right\rangle_1 \otimes \left|-2x + \frac{1}{3}\chi_2\right\rangle_2 \otimes \left|x - \frac{1}{2}\chi_1 + \frac{1}{3}\chi_2\right\rangle_3. \tag{27}$$

Its inverse transformation is

$$|p, \chi_1, \chi_2\rangle = \frac{1}{\sqrt{2\pi}} e^{-ip(\frac{1}{2}\chi_1 + \frac{1}{3}\chi_2)}\int_{-\infty}^{\infty} dx|x\rangle_1 \otimes |-2x + \chi_1 + \chi_2\rangle_2 \otimes |x - \chi_1\rangle_3 e^{ixp}, \tag{28}$$

which tells us that for the state $|p, \chi_1, \chi_2\rangle$ once particle 1 is measured in the state $|x\rangle_1$, particle 2 immediately collapses to the coordinate eigenstate $|-2x + \chi_1 + \chi_2\rangle_2$, while particle 3 immediately collapses to $|x - \chi_1\rangle_3$. So (28), manifestly exhibiting $|p, \chi_1, \chi_2\rangle$'s entanglement, is the standard Schmidt decomposition of $|p, \chi_1, \chi_2\rangle$.

On the other hand, by virtue of the expression of the momentum eigenstate

$$|p\rangle_j = \pi^{-1/4}\exp\left(-\frac{1}{2}p^2 + \sqrt{2}ipa_j^\dagger + \frac{1}{2}a_j^{\dagger 2}\right)|0\rangle_j, \quad j = 1, 2 \tag{29}$$

we can make a two-fold Fourier transformation to $|p, \chi_1, \chi_2\rangle$ in the following way:

$$\frac{1}{4\pi^2}\iint_{-\infty}^{\infty} d\chi_1\, d\chi_2\, e^{-i(u\chi_1 + v\chi_2)}|p, \chi_1, \chi_2\rangle$$
$$= \frac{1}{2\pi}\left|\frac{1}{6}p - v - u\right\rangle_1 \otimes \left|-\frac{1}{3}p - v\right\rangle_2 \otimes \left|\frac{1}{6}p - v + u\right\rangle_3. \tag{30}$$

The inverse transformation for (30) is

$$|p, \chi_1, \chi_2\rangle = \frac{1}{2\pi}\iint_{-\infty}^{\infty} du\, dv\, e^{i(u\chi_1 + v\chi_2)}\left|\frac{1}{6}p - v - u\right\rangle_1 \otimes \left|-\frac{1}{3}p - v\right\rangle_2 \otimes \left|\frac{1}{6}p - v + u\right\rangle_3. \tag{31}$$

Again we see that $|p, \chi_1, \chi_2\rangle$ is an entangled state. Equation (31) shows that once particle 1 is measured in the momentum eigenstate, the other two particles immediately collapse to their momentum eigenstates too no matter how far the distances between the three particles are.

4 Teleportation of a Bipartite Entangled State

We now attempt to apply $|p, \chi_1, \chi_2\rangle$ to discussing quantum teleportation of bipartite state of continuous variable. We shall show that the $|p, \chi_1, \chi_2\rangle$ representation can help us to recapitulate the theory of quantum teleportation of continuous variables concisely.

Let particles 3, 4 and 5 be prepared in an tripartite entangled state $|p, \chi_1, \chi_2\rangle_{345}$ (shared by Alice, Bob and Claire), while Alice initially possesses an unknown state $|E(q)\rangle_{12}$ of the form

$$|E(q)\rangle_{12} = \int_{-\infty}^{\infty} dx\, E(x)|x\rangle_1 \otimes |x-q\rangle_2, \tag{32}$$

thus the total initial state is

$$|\Psi\rangle = |E\rangle_{12} \otimes |p, \chi_1, \chi_2\rangle_{345}. \tag{33}$$

Alice wants to teleport this unknown state to Bob and Claire. For this purpose, Alice makes a specific measurement on particles 1, 2 and 3 with the projection basis being

$$\Pi_3 = |p'\rangle_{11}\langle p'| \otimes |\eta\rangle_{2323}\langle \eta|, \tag{34}$$

($|p'\rangle_1 \otimes |\eta\rangle_{23}$ can also be viewed as continuous Bell basis). According to (24) and (28), after the measurement the projected state for particles 4, 5 is

$$\begin{aligned}
&{}_{23}\langle \eta|_1\langle p'|E\rangle_{12} \otimes |p, \chi_1, \chi_2\rangle_{345} \\
&\quad = \frac{1}{2\pi} e^{-ip(\frac{1}{2}\chi_1 + \frac{1}{3}\chi_2)} e^{i\eta_2\eta_3/2} \int_{-\infty}^{\infty} dx\, e^{-ip'(x+q)} E(x+q) \\
&|-2x + 2\eta_2 + \chi_1 + \chi_2\rangle_4 \otimes |x - \eta_2 - \chi_1\rangle_5 e^{i(x-\eta_2)p} e^{-i\eta_3 x} \\
&\quad = M U_4 \otimes U_5 \otimes |E\rangle_{45}
\end{aligned} \tag{35}$$

where we have used

$$\langle x|p\rangle = \frac{1}{\sqrt{2\pi}} e^{ipx} \tag{36}$$

and defined

$$U_4 = \int_{-\infty}^{\infty} dx\, e^{-ip'x - i\eta_3 x} |-2x + 2\eta_2 + \chi_1 + \chi_2\rangle_{44}\langle x+q|, \tag{37}$$

$$U_5 = \int_{-\infty}^{\infty} dx\, e^{ixp} |x - \eta_2 - \chi_1\rangle_{55}\langle x|, \tag{38}$$

$$M = \frac{1}{2\pi} e^{-ip(\frac{1}{2}\chi_1 + \frac{1}{3}\chi_2)} e^{i\eta_2\eta_3/2} e^{-ip'q} e^{-i\eta_2 p}. \tag{39}$$

U_4 and U_5 are unitary operators, and M is a phase factor. Alice then informs Bob and Claire of her measurement outcomes (η and p') via classical channels, after receiving the classical information, Bob and Claire perform the unitary transformation U_4^{-1} and U_5^{-1} respectively. In this way they can have their particles in the entangled state as the state to be teleported. Thus the teleportation can be carried out successfully.

Using $|p, \chi_1, \chi_2\rangle$ we can also teleport a two-mode squeezed state [22]. From Ref. [23] we know that the two-mode squeezing operator can be neatly expressed in the EPR entangled state representation,

$$S_{12} = \int \frac{d^2\eta}{\mu\pi} \left|\frac{\eta}{\mu}\right\rangle_{1212} \langle \eta| \tag{40}$$

The two-mode squeezed vacuum state is

$$\begin{aligned}
&\int \frac{d^2\eta}{\pi}\left|\frac{\eta}{\mu}\right\rangle_{1212}\langle\eta|00\rangle \\
&\quad=\int \frac{d^2\eta}{\pi}e^{-|\eta|^2/2}\left|\frac{\eta}{\mu}\right\rangle_{12} \\
&\quad=\int \frac{d^2\eta}{\pi}\int_{-\infty}^{\infty}dx\exp\left(-\frac{|\eta|^2}{2}-\frac{i\eta_1\eta_2}{2\mu^2}\right)|x\rangle_1\otimes\left|x-\frac{\eta_1}{\mu}\right\rangle_2 e^{i\eta_2 x/\mu}.
\end{aligned} \tag{41}$$

Comparing (41) with (24) we see that the two-mode squeezed state is an entangled state too. Suppose Alice wants to teleport it to Bob and Claire, the total initial state is

$$|\Phi\rangle=\int \frac{d^2\eta}{\pi}\left|\frac{\eta}{\mu}\right\rangle_{1212}\langle\eta|00\rangle_{12}\otimes|p,\chi_1,\chi_2\rangle_{345}. \tag{42}$$

She operates a joint measurement as follows:

$$\begin{aligned}
&{}_{23}\langle\eta'|_1\langle p'|\int \frac{d^2\eta}{\pi}\left|\frac{\eta}{\mu}\right\rangle_{1212}\langle\eta|00\rangle_{12}\otimes|p,\chi_1,\chi_2\rangle_{345} \\
&=\frac{1}{2\pi^2}e^{-ip(\frac{1}{2}\chi_1+\frac{1}{3}\chi_2)}e^{i\eta_2'\eta_3'/2}\int d^2\eta\int_{-\infty}^{\infty}dx\,\exp\left(-\frac{|\eta|^2}{2}-\frac{i\eta_1\eta_2}{2\mu^2}\right)e^{-i\eta_3'x} \\
&\quad\times e^{-ip'(\frac{\eta_1}{\mu}+x)}e^{i\eta_2(\frac{\eta_1}{\mu}+x)/\mu}|-2x+2\eta_2'+\chi_1+\chi_2\rangle_4\otimes|x-\eta_2'-\chi_1\rangle_5 e^{i(x-\eta_2')p} \\
&=M'U_4'\otimes U_5'\otimes S_{45}|00\rangle_{45}
\end{aligned} \tag{43}$$

where

$$M'=\frac{1}{2\pi}e^{i\eta_2'\eta_3'/2}e^{-ip(\frac{1}{2}\chi_1+\frac{1}{3}\chi_2)}e^{-i\eta_2'p} \tag{44}$$

$$U_4'=\int_{-\infty}^{\infty}dx\,e^{-ip'(x+\frac{\eta_1}{\mu})}|-2x+2\eta_2'+\chi_1+\chi_2\rangle_{44}\left\langle x+\frac{\eta_1}{\mu}\right| \tag{45}$$

$$U_5'=\int_{-\infty}^{\infty}dx\,e^{-i(\eta_3'-p)x}|x-\eta_2'-\chi_1\rangle_{55}\langle x|. \tag{46}$$

Alice then sends her measurement outcomes to Bob and Claire by classical channels, Bob and Claire perform $U_4'^{-1}$ and $U_5'^{-1}$ respectively, and as a result the teleportation is achieved, the two-mode squeezed state $S_{45}|00\rangle_{45}$ is now shared by them.

In the above discussions we have shown the advantages of the usage of $|p,\chi_1,\chi_2\rangle$ state in three folds: (1) Teleported states can be calculated more explicitly, so one can reach straightly to the point. (2) Using the $|p,\chi_1,\chi_2\rangle$ representation the discussion of teleportation can be conveniently converted either to the coordinate-momentum representation or to the particle number representation. (3) This approach expounds teleportation theory in simple language by virtue of the $|p,\chi_1,\chi_2\rangle$ representation.

5 Conclusions

In summary, for the newly constructed tripartite entangled state representation $|p,\chi_1,\chi_2\rangle$, we discuss its entanglement by obtaining its Schmidt decomposition. Then, application of

$|p, \chi_1, \chi_2\rangle$ to quantum teleportation can be conveniently carried out. Thus, it is worth introducing the new three-mode entangle state representation $|p, \chi_1, \chi_2\rangle$.

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